

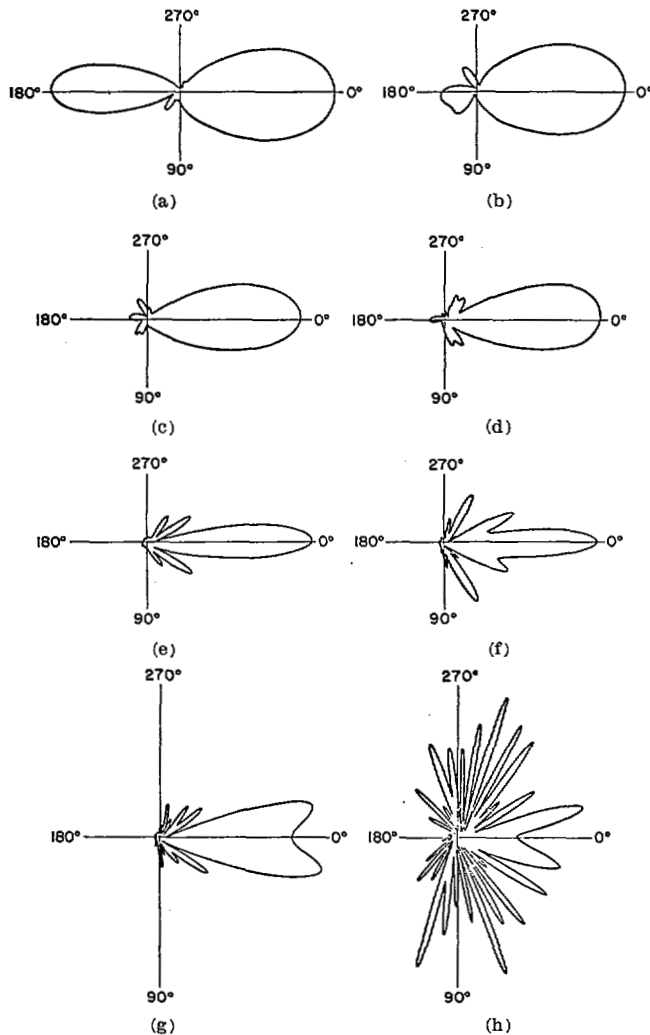


Fig. 2. Beam patterns of the quadrifilar helix antenna. Pitch angle = 35° , helix diameter = 3 in, length = 24 in, ground plane diameter = 10 in. (a) 550 MHz. (b) 650 MHz. (c) 900 MHz. (d) 1400 MHz. (e) 2000 MHz. (f) 2300 MHz. (g) 2600 MHz. (h) 3400 MHz.

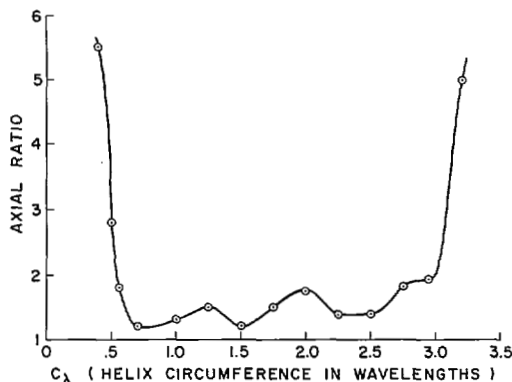


Fig. 3. Ellipticity of quadrifilar helix antenna.

occurs at 3400 MHz. The input VSWR was less than 1.40 between 300 MHz and 3000 MHz (for perfect hybrids and symmetrical helix antenna arms, the VSWR should be unity). The bandwidth for endfire patterns with no beam splitting, 3 dB or greater sidelobes, and axial ratio less than 3:1, is about 4:1.

Preliminary tests were also made on a quadrifilar helix antenna with a simplified feed system consisting of a single quadrature hybrid rather than the 3 hybrids used in the feed system of Fig. 1. Two adjacent windings were fed at 0° and 90° phase without

feeding the other two windings. The beam patterns are promising [7]; broad-band behavior is obtained although pattern breakup does occur at one frequency.

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Some Effects of Asymmetric Excitations on Dipole Radiators

Abstract—The recent model due to Shen, Wu, and King for the current distribution along the asymmetrically-driven thin cylindrical antenna is used in the calculation of current and far-field values for several cases of interest. The graphs show how the current distribution approaches the limiting sinusoidal assumption as the generator location is moved toward a free end. The antenna conductor thickness is shown to cause decaying sinusoidal variations on the radiation fields from the half-and full-wave dipoles having large excitation asymmetries. The model was found to yield accurate current amplitudes for dipoles at least 0.30λ in total lengths; however, the current phase is inaccurate for points closer than about 0.025λ from a free end. This latter value is also the restriction on how close the generator may be from the free end.

By considering the total axial current on a transmitting dipole to consist of the superposition of a single wave outgoing from the generator terminals and an infinite number of end-reflected waves, the following general expression for the current was proposed by Shen *et al.* [1]:

$$I(Z) = I_f(|H + Z|) + C_d I_f(L + Z) + C_u I_f(L - Z) \quad (1)$$

where I_f is the current on an infinitely long linear antenna, Z is the distance along the dipole of total length $2L$, H is the position of the generator terminals (to the left of the origin), and C_d and C_u are complex parameters accounting for fractional reflections at the free ends (left and right, respectively,) of the antenna. The dimensional variables Z , H , L , and a (the radius of the antenna conductor) are expressed in terms of the radiation wavelength λ . The nature of the reflection process is such that each wave appears to originate from a virtual voltage generator at the point of reflection. Even though C_d and C_u are those given by Shen *et al.* [1, eqs. (14), (15)], expressions for I_f other than theirs may be used instead [2], [3].

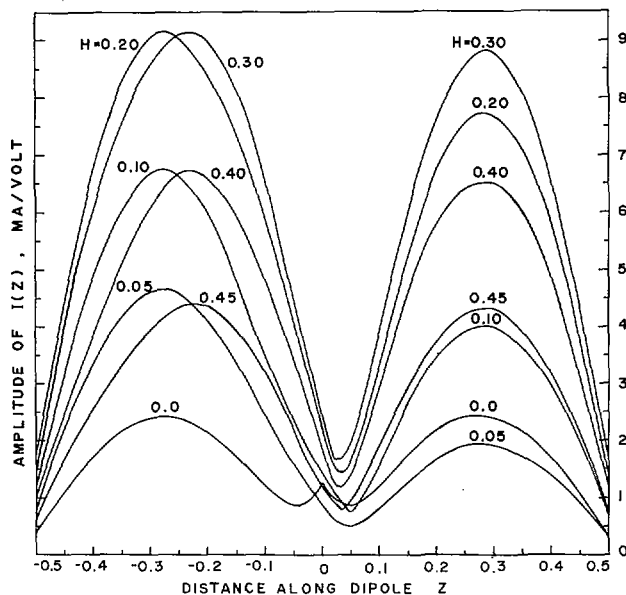


Fig. 1. Amplitude of current on full-wave dipole as excitation asymmetry varies.

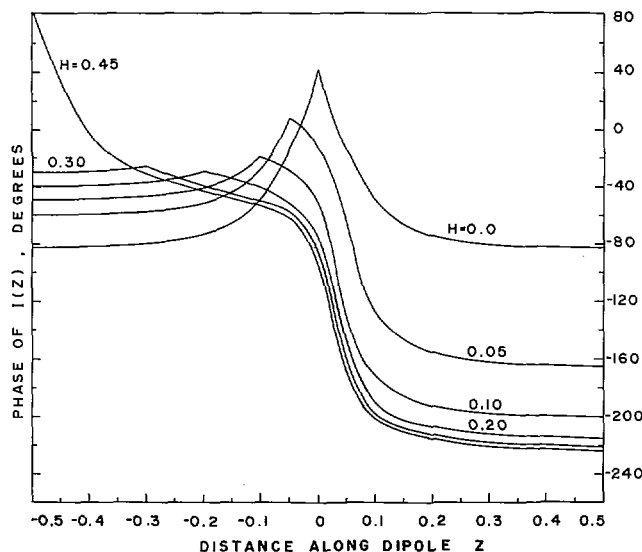


Fig. 2. Phase of current on full-wave dipole as excitation asymmetry varies.

The axial component of current on infinitely long linear antennas is expressible for positive Z in the form [1]

$$I_f(Z, t) = \frac{jV_0 \exp[-j(\omega t + KZ)]}{\eta_0} \ln \left(1 - \frac{j2\Pi}{C(Z) + j3\Pi/2} \right) \quad (2)$$

where

$$C(Z) = \ln [(Ka)^{-2}(KZ + [(KZ)^2 + e^{-2\gamma}]^{1/2})] - \gamma.$$

In these equations ω is the angular frequency in rad/s, t is the time in seconds, η_0 is the free-space impedance in ohms, $K = 2\Pi/\lambda$ rad/m is the Euler's constant, and V_0 is the peak value of the voltage generator in volts. By superposition these equations may be used for calculating the current distribution on multiply excited dipoles. They are also useful for determining the impedance at the driving point Z_i of the asymmetrically excited dipole

$$Z_i = \frac{V_0}{I(H)} \quad (3)$$

in which the factor $\exp(-j\omega t)$ is implied.

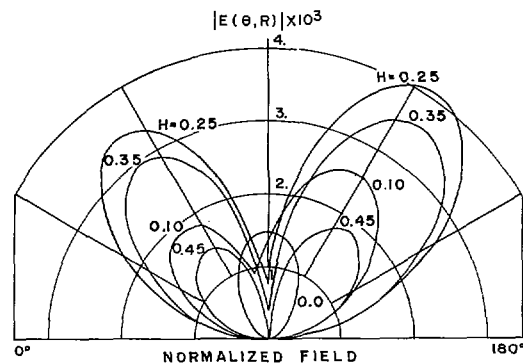


Fig. 3. Amplitude of far field for full-wave dipole.

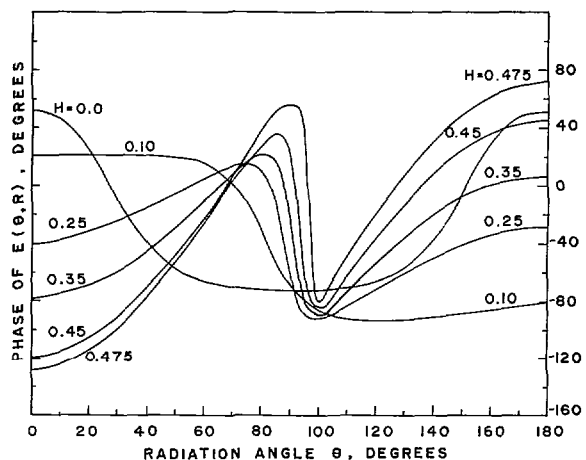


Fig. 4. Phase of far field for full-wave dipole.

Equations (1) and (2) were used in the generation of a family of graphs for the current amplitude and phase on the full-wave dipole as functions of the asymmetry H . Note in Figs. 1 and 2 that as the excitation approaches the left free end of the dipole both the amplitude and the phase (180° phase shift at center) tend toward the sinusoidal assumption for the end-driven case [4]. Figs. 1-4 are for $V_0 = 1$ V and $\alpha = 0.007$.

Once the current distribution on the antenna is available the electromagnetic fields in the far-field region are obtained straightforwardly with the use of the magnetic vector potential. For the thin dipole the polar component of the electric-field vector takes the integral form

$$E(\theta, R) = \frac{jK\eta_0 \exp(-jKR)}{4\Pi R} \cdot \sin \theta \int_{-L}^L I(Z, t) \exp[-j(\omega t - KZ \cos \theta)] dZ \quad (4)$$

where R is the radial distance in meters from the antenna center to a point in the far field and its polar angle θ . The preceding current expression was used in this expression in order to evaluate the far fields of the half- and full-wave dipoles for several excitation asymmetries. The results of numerical calculations are shown in Figs. 3 and 4, which were normalized by the factor

$$N(R, \lambda) = \frac{K\eta_0 V_0}{4\Pi R} \angle \left(\frac{\Pi}{2 - KR} \right). \quad (5)$$

Note that when the excitation asymmetry approaches the end-driven condition, the field amplitude shows two asymmetric lobes and the field phase indicates a trend for a 180° phase shift between them. The limiting condition of two identical lobes, 180° out of phase, is predictable with the sinusoidal-current assumption for infinitely thin dipoles. The far-field amplitude of the half-wave

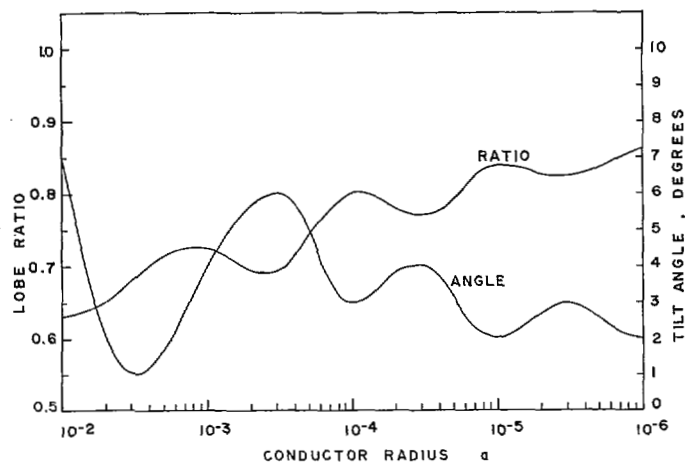


Fig. 5. Effects of conductor thickness on tilt angle of half-wave dipole ($H = 0.225$) and on lobe ratio of full-wave dipole ($H = 0.475$).

dipole showed an increasing tilt angle in the direction away from the direction of the increasing excitation asymmetry. The existence of the tilt on a field pattern of asymmetrically excited dipoles is attributable to the traveling-wave component in the current distribution along the antenna.

The effect of the conductor thickness on the tilt angle (as measured from broadside) for the field pattern of the asymmetrically excited ($H = 0.225$) half-wave dipole is shown in Fig. 5. The behavior of the tilt angle is that of an exponentially decaying sinusoid which approaches zero as the conductor thickness approximates the "infinitely thin" assumption. The other curve in this figure represents the ratio of the peak power in the lobe nearest to the excitation point to the peak power in the other lobe for the asymmetrically excited ($H = 0.475$) full-wave dipole. The sinusoidal prediction of unity ratio is slowly approached as the thickness of the conductor decreases.

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On the Focal Plane Distribution of an Imperfect Paraboloid

Abstract—An asymptotic expansion has been obtained for the focal plane distribution (or the radiation pattern) of aperture antennas with Gaussian distributed random phase deviations. This asymptotic series not only offers convenience in numerical computation for large phase errors, but also clarifies the region of validity for the geometrical optics approximation.

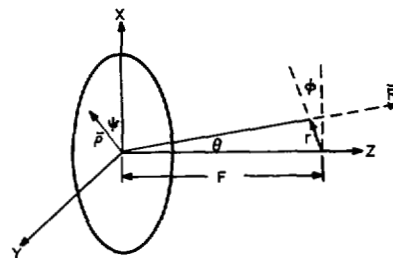


Fig. 1. Sketch of geometry.

The focal plane distribution of a thin lens or a shallow paraboloidal reflector with random surface deviations is of crucial importance in determining the required detector size of the incoherent optical receiver with a large imperfect collector [2]. The focal plane distribution in the vicinity of the focus is mathematically equivalent to the far-field radiation pattern. The scalar Kirchhoff approximation has been often used as a starting point for this problem [1]. Assuming Gaussian distributed phase deviations, Ruze [1] obtained a power series solution which has been evaluated for $x \leq 4$, where x is the mean-square phase error. When the phase error is very large ($\gg 1$ rad), an approximate solution is available either as an asymptotic limit [3], [5] or through geometrical optics arguments [4]. This communication will present an asymptotic expansion of the solution for the Gaussian model. The asymptotic series not only offers convenience in numerical computation for larger values of x , but also distinguishes between wave optics and geometrical optics solutions and thereby clarifies the region of validity for the geometrical optics solution.

The far-field pattern of a circular aperture is

$$G(\theta, \phi) = \frac{4\pi}{\lambda^2} \frac{\left| \int f(\bar{p}) \exp(jk\hat{R} \cdot \bar{p}) \exp(j\delta(\bar{p}, \psi)) dS \right|^2}{\int f^2(\bar{p}) dS} \quad (1)$$

where the direction vector $\hat{R} = \bar{R}/R$ and the aperture position vector $\bar{p}$ are shown in Fig. 1; $f(\bar{p})$ is the illumination function and $\delta(\bar{p}, \psi)$ is the phase error. Writing the numerator as a double surface integral of two running vector variables $\bar{p}_1$ and $\bar{p}_2$ and assuming a Gaussian covariance function $\bar{\delta}^2 \exp(-|\bar{p}_1 - \bar{p}_2|^2/c^2)$ for the random phase deviations, the expected value of the gain will be

$$\overline{G(\theta, \phi)} = \frac{4\pi}{\lambda^2} \iint \Phi(\bar{\tau}) \exp[jk\tau \sin \theta \cos(\psi - \phi)] \cdot \exp\left[-x \left(1 - \exp\left(\frac{-\tau^2}{c^2}\right)\right)\right] \tau d\tau d\psi \quad (2)$$

where $x = \bar{\delta}^2$, c is the correlation length, $\tau = |\bar{\tau}| = |\bar{p}_1 - \bar{p}_2|$, and $\Phi(\bar{\tau})$ is the illumination correlation function

$$\Phi(\bar{\tau}) = \frac{\int f(\bar{p}_1)f(\bar{p}_1 + \bar{\tau}) dS_1}{\int f^2(\bar{p}_1) dS_1}$$

The radiation pattern becomes the focal plane distribution if $\sin \theta$ in (2) is replaced by r/F as illustrated in Fig. 1. Expanding the exponential function $\exp[x \exp(-\tau^2/c^2)]$ into a power series identifies the first term of the integral in (2) as the unperturbed pattern. Since the illumination function remains essentially constant within the correlation length c which is assumed small compared with the aperture dimensions and the principal contribution of the integral comes from $\tau < c$, $\Phi(\bar{\tau})$ may be assumed to be unity in evaluating the remaining terms, and the limit of integration for τ can be extended to infinity. These arguments